Dr. M.K.K Arya Model School

HOLIDAY ASSIGNMENT

Subject-Maths

Class-XI

1) For all sets A,B,C prove that $(A-B) \cap (C-B) = (A \cap C) - B$
2) Prove that $A - (B \cap C) = (A-B) \cup (A-c)$ by using properties of set.
3) Describe the following sets by roster method :

   (i) { x:x is a letter of the word LITTLE } (ii) $\{ x:x<40, x \in N \}$

4) Describe the following sets by property method :

   (i) { 5,10,15,20 } (ii) { 1,4,9,16,25 }

5) Show that the set of letter needed to spell "CATARACT" and the set of letters needed to spell "TRACT" are equal.
6) Find the smallest set Y such that $Y \cup \{1,2\} = \{1,2,3,5,9\}$.
7) If A ={ 4,5,7,8,10 } , B={ 4,5,9 } and C = { 1,4,6,9},then verify that :
   (i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   (ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
8) Find A-B and B-A if :
   (i) A={ x:x>4, x<2 }, B= { 6,7,10,15 } (ii) A={ 5,7,9,11,13,15 }, B={ $x:x=2n+1, 2 \leq n \leq 7, n \in N$ }
9) 7. Let X = { 1,2,3,....,12 }. A = { 4,5,9,11 } is a subset of B={ 1,2,4,5,8,9 }. Verify that B′ is a subsetof A′.
10) If X = { 1,2,3,.....11 },A = { 2,5,9,10 }, B={ 1,4,7,9 }, then verify that:
    (i) $(A \cap B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$
11) Prove that :

    (i) $A-(B \cap c) = (A-B) \cap (A-c)$

    (ii) (ii) $A-(B \cap C) = (A-B) \cup (A-C)$

12) In a class of 60 boys, each playing at least one game, there are 45 boys who play cards and 30 boysplay. Using set theory, find :

(i) How many boys play both games ?

(ii) How many boys play cards only ?

(iii) How many boys play carrom only ?

13) A college awarded 38 medals in Football, 15 in Basketball and 20 in Cricket. If these medals went to a total of 58 men and only 3 men got medals in all the 3 sports , how many received medals in exactly two of the three sports ?

14) Find A **Δ** B, if :

(i) A= { 4,5,7 }, B= { 5,6,8 } (ii) A={ 3,4,6 }, B= **φ**

15) Prove that (A-B)∪(B-A) = **ϕ** ⇔ A=B

16) If A and B are sets, then prove that A-B,A∩B and B-A are pairwise disjoint sets.

17) Prove that A∩(A∪B) = A

18) Prove that A∪B = A∩B iff A=B.

19) Let A and B be sets. If A∩X=B∩X=**ϕ** and A∪X = B∪X for some set X. Show that A = B.

20) Give an example to show that if A∪B and A∩B are given, then A and B are not uniquely determinable.

21) If A= { 4,5,8,12 }, B ={ 1,4,6,9 } and C= { 1,2,4,7,8,10 }, then find :A-(B-A)

22) In a survey of 60 people, it was found that 25 people read Newspaper H, 26 read Newspaper T, 26 read Newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three Newspaper. Find :

(i) the number of people who read atleast one of the newspapers

(ii) the number of people who read exactly one newspaper.

23) A survey of 500 television viewers produced the following information ; 285 watch football, 195 watch hockey, 115 watch basketball, 45 watch football and basketball, 70 watch football and hockey, 50 watch hockey and basketball, 50 do not watch any of the three games. How

many watch all the three games? How many watch exactly one of the three games?

24) State De-Morgan's Laws. Prove any one of them.

25) For any sets A and B, prove that $P(A \cap B) = P(A) \cap P(B)$ In a class of 60 students, 23 play Hockey, 15 play Basketball and 20 play Cricket. 7 play Hockey and Basketball, 5 play Cricket and Basketball, 4 play Hockey and Cricket and15 students do not play any of these games. Find how many play Hockey and Cricket but not Basketball without using venn Diagram?

26) Find the range of $f(x) = |x-3|$

27) A relation R is defined on the set Z of integers as: $(x,y) \in R \Leftrightarrow x2 + y2 = 25$

28) Find the domain of $f(x) = \sqrt{4-x2}$

29) If $f: R \to R$ is defined by $f(x) = x/x2+1$, find $f(f(2))$.

30) Let $f(x) = x+1$, $g(x) = x2-1/x-1$. Is $f=g$?

31) Find the domain of the function $f(x) = \sqrt{4-x} + 1/\sqrt{x2-1}$

32) Find the domain and range of function $f = \{(x:1/1-x2):x \in R,\ x \neq \pm 1\}$

33) Let A={1,2,3.....14}. Define a relation R on A by $R=\{(x,y):3x-y =0$, where $x,y \in A\}$. Depict this relationship using an arrow diagram. Write down its domain, codomain and range.

34) Find the domain and range of a function $f(x) = \sqrt{x^2-3x+2}$.

35) Draw the graph of the following function : $f(x) = x^2-4/x+2$

36) 11. $1/1.2.3 + 1/2.3.4 + 1/3.4.5 + ... + 1/n(n+1)(n+2) = n(n+3)/4(n+1)(n+2)$, $n \in N$.

37) If $f: R \to R$ is defined by $f(x) = x^2-3x+2$, find $f(f(x))$. Also evaluate $f(f(5))$.

38) Find the domain and range of $F(x) = 3/2-x^2$

39) Let $f = \{(x, x^2/1+x^2), x \in R\}$ be a function from R into R. Determine the range of f.

40) Prove that $:1+1/4+1/9+1/16+.....+1/n^2 < 2-1/n$, $n(>1) \in N$, by using P.M.I.

41) Prove by induction that $n^5/5+n^3/3+7n/15$ is a natural number for all $n \in N$.

42) Prove by mathematical induction for all natural numbers n, $a^{2n-1}-1$ is divisibleby a-1.

43) Prove by induction that for all n∈N :
(1+3/1)(1+5/4)(1+7/9)...(1+2n+1/n2) = (n+1)2.

44) Use induction to show that $(25)^{n+1}$ -24n + 5735 isi divisible by $(24)^2$ for n = 1,2,3,......

45) By induction, prove that $\{(41)^n - (14)^n\}$ is a multiple of 27.

46) Prove that $n^{11}/11 + n^5/5 + n^3/3$ +62/165n is a positive integer for all n∈N.

47) Prove by induction the sum of cubes of three consecutive natural number is divisible by 9.

48) Prove that 1/n+1 + 1/n+2 +.......+1/2n > 13/24.

49) Prove by induction : $a + ar + ar^2 + \ldots ar^{n-1} = a(r^{n-1}/r-1), r \neq 1$.

50) If tan A - tan B = x and cot B - cot A = y, prove that cot (A - B) = 1/x +1/y.

51) If 3 tan A tan B = 1, prove that 2cos(A + B) = cos(A - B)

52) If cos(**α** - **β**) + cos(**β** - **γ**) + cos (**γ** - **α**) = -3/2, prove that cos**α** + cos**β** + cos**γ** = sin**α** + sin**β** + sin**γ** = 0.

53) Prove that sinx/cos3x + sin3x/cos9x + sin9x/cos27x = ½(tan 27x - tanx).

54) If sin(x+y)/sin(x-y) = a+b/a-b , show that tanx/tany = a/b.

55) If tanx = sin**α** - cos**α**/sin**α** + cos**α** , then show that sin**α** + cos**α** = √2cosx.

56) If **α** and **β** are two solutions of the equation atanx + bsecx = c , then find the values of sin(**α** - **β**) and cos (**α** + **β**).

57) Prove that sin A (1 + tan A) + cos A (1 +cot A) = sec A + cosec A.

58) Prove that (cosec**θ** - sec**θ**) (cot**θ** - tan**θ**) = (cosec**θ** + sec**θ**) (sec**θ** cosec**θ** - 2).

59) Prove that (1 + cot A+ tan A) (sin A - cos A) = sec A/cosec^{2}A - cosec A/sec^{2}A.

60) Prove that cos^{2}A + cos^{2}B - 2 cos A cos B cos(A + B) = sin^2(A+ B)

61) Prove that 1 + tan**θ** tan**θ**/2 = sec**θ** = tan**θ** cot**θ**/2 -1

62) Prove that $\tan 50° = 2 \tan 10° + \tan 40°$.

63) If $\tan(a+\theta) = n \tan(a-\theta)$, show that $\sin 2\theta/\sin 2\alpha = n-1/n+1$.

64) If α and β are solutions of the equation $a \tan\theta + b \sec\theta = c$, show that $\tan(\alpha+\beta) = 2ac/a2-c2$.

65) Evaluate $\sin A \sin(B+C) - \cos A \cos(B+C)$.

66) If $\cos(\alpha-\beta) + \cos(\beta-\gamma) + \cos(\gamma-\alpha) = (-3/2)$, prove that $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$.

67) Prove that $(\sin A + \sin 3A + \sin 5A + \sin 7A)/(\cos A + \cos 3A + \cos 5A + \cos 7A) = \tan 4A$

68) If $\cos(\theta + 2\alpha) = n \cos\theta$, show that $\cot\alpha = 1+n/1-n \tan(\theta+\alpha)$.

69) Prove that $\cos 5A+\cos 7A/\sin 5A+\sin 7A = \cot 6A$.

70) If $\cos(A-B) = 3 \cos(A+B)$, show that $\cot A \cot B = 2$

71) Prove that $\sec 2A + \tan 2A = (\cos A+\sin A)/(\cos A-\sin A) = \tan(\Pi/4 + A)$.

72) If $\cos(A+B) \sin(C-D) = \cos(A-B) \sin C+D)$, prove that $\tan A \tan B \tan C + \tan D = 0$.

73) Prove that $\cos 6° \cos 42° \cos 66° \cos 78° = 1/16$

74) If $\operatorname{cosec} A + \sec A = \operatorname{cosec} B + \sec B$, prove that $\tan A \tan B = \cot(A+B)/2$.

75) Prove that $\cos^2 A + \cos^2(A+120°) + \cos^2(A-120°) = 3/2$

76) If $\tan A = ½$, $\tan B = 1/3$, prove that $\tan(2A + B)=3$

77) Prove that $\sin A(1+\tan A) + \cos A(1+\cot A) = \sec A + \operatorname{cosec} A$

78) Prove that $(\operatorname{cosec}\theta-\sec\theta)(\cot\theta-\tan\theta) = (\operatorname{cosec}\theta+\sec\theta)(\sec\theta \operatorname{cosec}\theta-2)$

79) If $\cos^2\alpha-\sin^2\alpha = \tan^2\beta$ then show that $\tan^2\alpha = \cos^2\beta-\sin^2\beta$

80) Prove that $\sin^8\theta-\cos^8\theta = (\sin^2\theta-\cos^2\theta)(1-2\sin^2\theta\cos^2\theta)$